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NATURAL CONVECTION MASS TRANSFER ADJACENT TO HORIZONTAL PLATES

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Abstract—Experiments on natural convection adjacent to horizontal plane surfaces were performed using the naphthalene sublimation technique. By analogy, the present mass transfer experiments correspond to heat transfer at a heated isothermal upward-facing plate or at a cooled downward-facing plate. Circular, square, and 7:1 rectangular planforms were employed in the tests. Overall mass transfer coefficients and Sherwood numbers were determined and correlated as a function of the Rayleigh number. A common correlation for all three planforms was attained by using characteristic lengths equal to the ratio of the surface area to the perimeter. The transfer coefficients predicted by laminar boundary layer theory fell well below the experimental data and exhibited a different dependence on the Rayleigh number. The oft-quoted empirical correlation of Fishenden and Saunders was found to be a less satisfactory representation of the present data than that of Mikheyev. In the range of lower Rayleigh numbers, the results given by a numerical finite-difference solution were very much lower than the experimental data.

NOMENCLATURE

A,	surface area;
D,	binary diffusion coefficient;
g,	acceleration of gravity;
h,	overall heat transfer coefficient;
h_m,	overall mass transfer coefficient;
k,	thermal conductivity;
L,	characteristic length;
L^0,	conventional characteristic length;
L^*,	characteristic length, A/P;
M_n,	molecular weight of naphthalene;
M_a,	molecular weight of air;
$\dot{m}$,	mass transfer rate;
Nu,	Nusselt number, hL/k;
P,	perimeter encompassing plate surface area;
Pr,	Prandtl number;
p_n,	partial pressure of naphthalene;
p_t,	total pressure;
$\bar{R}$,	universal gas constant;
Ra,	Rayleigh number, $[g\beta(T_w - T_\infty)L^3/\nu^2]Pr$;
Ra_m,	Rayleigh number for mass transfer, $[g(\rho_w - \rho_\infty)L^3/\rho_\infty\nu^2]Sc$;
Sc,	Schmidt number;
Sh,	Sherwood number, h_mL/D;
T,	absolute temperature;
W_n,	mass fraction of naphthalene, ρ_n/ρ;
β,	thermal expansion coefficient;
ρ,	mixture density;
ρ_a,	density of air;
ρ_n,	density of naphthalene vapor;
ν,	kinematic viscosity.

Subscripts

w,	at the subliming surface;
∞,	in the surroundings.

Superscripts

0,	based on length L^0;
*,	based on length L^*.

INTRODUCTION

THIS paper describes experiments on natural convection mass transfer adjacent to horizontal plane surfaces having various planforms. The experiments were performed using downward-facing plates of solid naphthalene situated in an air environment. Under the conditions of the tests, naphthalene vapor sublimes from the plate surface, thereby creating a binary boundary layer. Within the boundary layer, concentration-induced density differences give rise to buoyancy forces which create and sustain the natural convection motion. Since naphthalene vapor is heavier than air, a natural convection plume issues downward from the plate surface.

The just-discussed natural convection mass transfer phenomena are analogous to the natural convection heat transfer phenomena adjacent to a heated horizontal plate facing upward or a cooled horizontal plate facing downward. The validity of the analogy is favored by the low rates of sublimation mass transfer and the small concentrations of naphthalene vapor that are encountered in the experiments. The analogous thermal boundary condition is uniform surface temperature, corresponding to the uniform surface concentration of the naphthalene vapor. In view of the analogy, the Sherwood number results of the experiments can be interpreted as Nusselt number results.

There are a number of advantages in using a sublimation mass transfer system, in lieu of a heat transfer system, for determining transfer coefficients. One of these is that extraneous heat losses, either by conduction through supports and instrumentation or by radiation, are avoided. Such extraneous losses can well be of the same magnitude as the natural convection heat transfer. A second advantage is that the mass transfer system avoids the uncertainties associated with possible preheating of the fluid which may occur along the lateral edges of the heat transfer system.

The present investigation encompassed three plate planforms: square, 7:1 rectangle, and circle. One of the objectives of the research was to explore and correlate the effects of plate planform shape. A second objective was to obtain reliable results for the transfer coefficient over a wide range of Rayleigh numbers.

A survey of the literature reveals a number of publications dealing with natural convection heat transfer from horizontal plane surfaces. Those publications concerned with heated upward-facing plates or cooled downward-facing plates will be discussed here. Correlations involving the Nusselt and Rayleigh numbers are given by Fishenden and Saunders [1], Bosworth [2], and Mikheyev [3]. All that is known about the often-quoted Fishenden and Saunders correlation is that it is based on experiments involving square or nearly square plates situated in air, with surface-to-fluid temperature differences as high as 1000°F. No other information about the apparatus and measurement technique is available. Bosworth gives no background information about the basis of his correlation, neither with respect to plate shape or to the fluid environment; furthermore, there is some uncertainty as to which dimension of the plate is to be employed in applying the correlating equation. Similarly, the basis of Mikheyev's correlation is not described. The characteristic dimension in the correlating equation is specified as "the smaller side of the plate," which implies a rectangular planform.

Experiments involving a heated upward-facing horizontal plate were performed by Hassan and Mohamed [4] as a special case of a broader study which was concerned with inclined plates. Heat transfer coefficients were reported only for the central region of the plate, so that no direct comparison can be made with the overall heat transfer results of the present investigation. Husar and Sparrow [5] studied the flow patterns adjacent to horizontal heated plates of various planforms with the aid of an electrochemical flow visualization technique.

The only prior experiments known to the authors on natural convection mass transfer from unshrouded horizontal surfaces are those of Wragg [6] and of Wragg and Loomba [7], who used circular disks. The experiments were carried out using an electrochemical technique. The Schmidt numbers were on the order of 2300, which is about one thousand times larger than the Schmidt number of the present experiments. Furthermore, the Rayleigh numbers of Wragg's tests were, in the main, substantially higher than those encountered here.

The published analyses for natural convection adjacent to heated upward-facing plates or cooled downward-facing plates deal with highly elongated rectangular planforms, that is, strips of finite width and infinite length. In all cases, the thermal boundary condition was uniform wall temperature. A laminar boundary layer model was generally employed [8–13], with a finite difference formulation having been used for the low Rayleigh number range [14]. Relevant comparisons between the results of analysis and experiment will be made later.

EXPERIMENTAL APPARATUS

The design of the experiment was focused on three main objectives. These were: (a) To establish a controlled, quiescent environment where sublimation of naphthalene could take place at a constant rate for a period of time long enough to permit accurate measurements of the weight loss of the test specimen. (b) To devise a technique for performing highly accurate measurements of specimen weight loss at any time during a data run without disturbing the natural convection mass transfer process. (c) To prepare test specimens having very smooth mass transfer surfaces that were free of physical or chemical contamination.

The experimental apparatus was situated in a windowless isolation room that had thermally insulating walls about 30 cm thick. The door to the room was replaced by a $7\frac{1}{2}$ cm thick styrofoam door fitted with two plastic windows through which remote observations of instrumentation were made from a vantage point outside the isolation room. As described shortly, styrofoam baffles were placed in the neighborhood of the apparatus as a precaution against stray air currents. Owing to the large volume of the isolation room ($\sim 110\ \mathrm{m}^3$), the mass fraction of the naphthalene vapor in the room was only about 10^{-6} at the conclusion of a typical data run during which 100 mg of solid naphthalene had sublimed. In view of this, and since the mass fraction of the naphthalene vapor at the plate surface was approximately 5×10^{-4}, it follows that the accumulation of naphthalene vapor in the room during the course of a data run did not influence the mass transfer rate.

A schematic diagram of the test set-up is presented in Fig. 1. The figure shows a naphthalene test specimen, in the form of a horizontal plate, suspended beneath an analytical balance. The balance, Mettler Model B5 "Gram-atic," was fitted with a special harness to facilitate below-the-balance weighing. The harness was designed so that the test specimen was situated about one

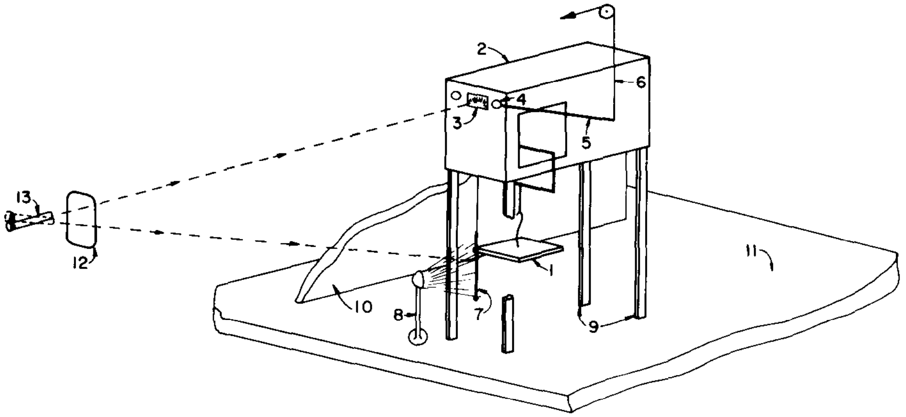

FIG. 1. Schematic diagram of the test set-up. 1: test specimen; 2: analytical balance; 3: optical readout; 4: 0·1 gram knob; 5: remote adjustment lever; 6: remote adjustment string; 7: thermometer; 8: thermometer light; 9: balance supports; 10: typical styrofoam baffle; 11: styrofoam floor; 12: plastic window in styrofoam door; 13: surveyor's telescope.

meter below the balance. Instantaneous values of the mass of the test specimen could be read to within 0·05 mg, which was also the stated accuracy of the balance. The accuracy of the balance readings was verified with the aid of a set of NBS class "S" weights.

The mass measurements indicated by the balance were read with the aid of a surveyor's telescope situated outside the isolation room. The telescope was set up to look through the aforementioned observation windows. In addition, a precision thermometer located near the test specimen was read remotely by means of the telescope, the readings being taken to the nearest 0·05°C. The indicating scale of the balance and the thermometer were illuminated only during the brief periods when readings were being made, thereby minimizing convection currents induced by heat from the instrument lamps. These lamps, 9 and 5 W respectively, were controlled from outside the isolation room.

Since the fine graduated scale of the balance only covered a range of 115 mg, it was sometimes necessary to make scale changes during the course of a data run. Such changes were accomplished remotely by means of a lever attached to the 0·1 g knob of the balance.

The floor beneath the analytical balance was covered with $7\frac{1}{2}$ cm thick styrofoam insulation, and vertical baffles of the same styrofoam material were placed around the area where the test specimen was suspended. The baffles were all at least 0·7 m away from the test specimen, and the distance from the specimen to the floor was about one meter. The function of the baffles was to shield against stray air currents. The area above the test specimen was open, except for the platform supporting the balance.

With regard to the preparation of the test specimens, a number of techniques was explored. An account of this work is given elsewhere [15], as are the fine details of the technique that was finally adopted.

In essence, the specimens were prepared by casting them in multiple-piece metal molds which consisted of a highly polished aluminum base plate on which were placed bars or plates which formed the sides of the mold. Reagent grade naphthalene ($C_{10}H_8$, molecular weight = 128·17) was heated in a glass beaker to about 180°C and then poured to a thickness of 0·5 cm into a mold that had been pre-heated to about 60°C. The naphthalene and the mold were allowed to cool naturally to room temperature. During the cooling process, a wire assembly was embedded in the solidifying naphthalene to facilitate suspension of the test specimen beneath the balance. For the larger specimens, reinforcing rods were inserted during solidification.

Removal of the cast specimen from the mold required considerable care and was accomplished by sharp hammer blows against the mold sides [15]. Upon removal from the mold, the top face and the sides of the specimen were sealed off with plastic wrap, so that the only exposed face was that which had contacted the base of the mold. This was the only face which participated in the natural convection mass transfer process.

The test specimens employed in the experiments included square, 7:1 rectangular, and circular planforms. The sides of the squares ranged from 1·27 to 20·2 cm, the short sides of the rectangles from 2·03 to 5·84 cm, and the diameters of the circles from 1·27 to 20·3 cm.

The embedded suspension wire was carefully adjusted so that the test specimen would hang with its exposed lower face horizontal. Prior to installation in the test apparatus, each specimen was suspended for eight hours in the laboratory immediately adjacent to the isolation room. This period was allowed to permit thermal equilibrium to be established and stray deposits of naphthalene to sublime away from the outer surface of the plastic wrap.

All data runs were made with the isolation room completely sealed off. Tests indicated that the influence of any initial room disturbances and of the starting transient died away after about 40 min of operation. The duration of a data run depended on the size of the test specimen, with a typical run lasting about 10 h. After completion of a run, the isolation room was purged for several hours with the aid of a large circulating fan. Thereafter, a period of one or two days was allowed for thermal equilibrium to be established.

The data collected during the course of a run included the instantaneous mass of the naphthalene test specimen, and the corresponding environment temperature, barometric pressure, and time. All told, there were 62 final data runs. Auxiliary tests were also made using naphthalene test specimens in which thermocouples had been embedded.

RESULTS AND DISCUSSION

Data analysis

Natural convection mass transfer results will be presented in terms of a mass transfer coefficient h_m defined as

$$h_m = \frac{\dot{m}/A}{\rho_w(W_{nw} - W_{n\infty})}. \tag{1}$$

The mass transfer rates $\dot{m}$ were determined from the measured changes in test specimen mass during measured intervals of time, and A denotes the area of the subliming surface. Inasmuch as $\dot{m}$ represents the mass transfer rate for the surface as a whole, then, correspondingly, h_m is an overall transfer coefficient. The mass fraction W_n of the naphthalene vapor is given by

$$W_n = \rho_n/\rho, \quad \rho = \rho_n + \rho_a \tag{2}$$

where ρ_n, ρ_a and ρ respectively represent the local densities of naphthalene, air, and the naphthalene–air mixture. The subscripts w and ∞ denote conditions at the plate surface and in the environment. In the present experiments, the mixture density ρ was very nearly constant across the boundary layer, so that the ρ_w appearing in the denominator of equation (1) could be replaced by ρ_∞ or by a mean value without altering h_m.

Since the mass fraction of naphthalene in the environment was negligible compared with the mass fraction at the wall (5×10^{-4} compared with 10^{-6} or less), $W_{n\infty}$ can be omitted from equation (1). Consequently, the denominator of equation (1) becomes, with the aid of the perfect gas law,

$$\rho_w W_{nw} = \rho_{nw} = p_{nw} M_n/\bar{R}T. \tag{3}$$

Under the condition that equilibrium prevails between the solid and vapor phases of naphthalene at the plate surface, the vapor pressure p_{nw} can be determined from the surface temperature. In this connection, auxiliary experiments revealed the absence of measurable temperature differences (at least to within 0·05°C) between the plate surface and the environment.

There are several correlations appearing in the literature for the vapor pressure–temperature relation for naphthalene. A careful evaluation by the present authors led to the conclusion that the most reliable among these are the representations given by Sogin [16] and by Christian and Kezios [17]. A comparison of the two correlations at a typical test temperature of 23°C showed only a 0·15 per cent difference in p_{nw}; at 26°C, the difference was 0·72 per cent. The Sogin correlation

$$\log_{10}(p_{nw}) = 11{\cdot}884 - 6713/T_w \tag{4}$$

was adopted for the data reduction. In equation (4), p_{nw} is in lb/ft^2 and T is in ^{0}R.

Mass transfer coefficients can be evaluated by means of equations (1), (3) and (4). A dimensionless representation of the results was made in terms of the Sherwood number Sh, given by

$$Sh = h_m L/D \tag{5}$$

where L is a characteristic dimension and D is the binary diffusion coefficient. The substitution of equations (1) and (3) into equation (5) yields

$$Sh = \frac{(\dot{m}/A)\bar{R}TL}{\nu p_{nw} M_n} . Sc \tag{6}$$

where Sc denotes the Schmidt number, ν/D, which was assigned a value of 2·5 [16]. Equation (6) was employed to calculate Sherwood numbers, using the experimental data as input.

The Sherwood number results are to be

presented as a function of the mass transfer Rayleigh number Ra_m, which is defined here as

$$Ra_m = \frac{g(\rho_w - \rho_\infty)L^3}{\rho_\infty \nu^2} . Sc. \qquad (7)$$

By making use of Dalton's law and of the perfect gas law, the quantity $(\rho_w - \rho_\infty)/\rho_\infty$ becomes $(p_{nw}/p_t)\,(M_n - M_a)/M_a = 3{\cdot}43\,(p_{nw}/p_t)$, where p_t is the total pressure. Consequently, in terms of measured quantities, Ra_m is

$$Ra_m = \frac{3{\cdot}43 g p_{nw} L^3}{p_t \nu^2} . Sc. \qquad (8)$$

The kinematic viscosity appearing in equations (8) and (6) was evaluated as that for pure air, which is justifiable since the maximum concentration of naphthalene was very small.

The Rayleigh number is used as the correlating parameter instead of the Grashof number because it minimizes the separate dependence of the Sherwood (or Nusselt) number on the Schmidt (or Prandtl) number. This approach has already been employed in [1–3, 6, 7] to correlate natural convection heat and mass transfer data.

It still remains to specify the characteristic lengths L which appear in equations (6) and (8). Two groups of characteristic lengths will be employed here. In the first group, L is taken as: (a) the side of a square plate, (b) the short side of a rectangular plate, and (c) the diameter of a circular plate. These characteristic lengths have been employed by prior investigators and will be termed conventional lengths and denoted by L^0. Results based on the conventional lengths will be presented first, after which an alternative presentation will be made on the basis of a second group of characteristic lengths.

Presentation of results

The Sherwood number results of the present investigation are plotted in Fig. 2 as a function of the Rayleigh number, with L evaluated in terms of the just-described conventional characteristic lengths L^0. The data symbols are identi-

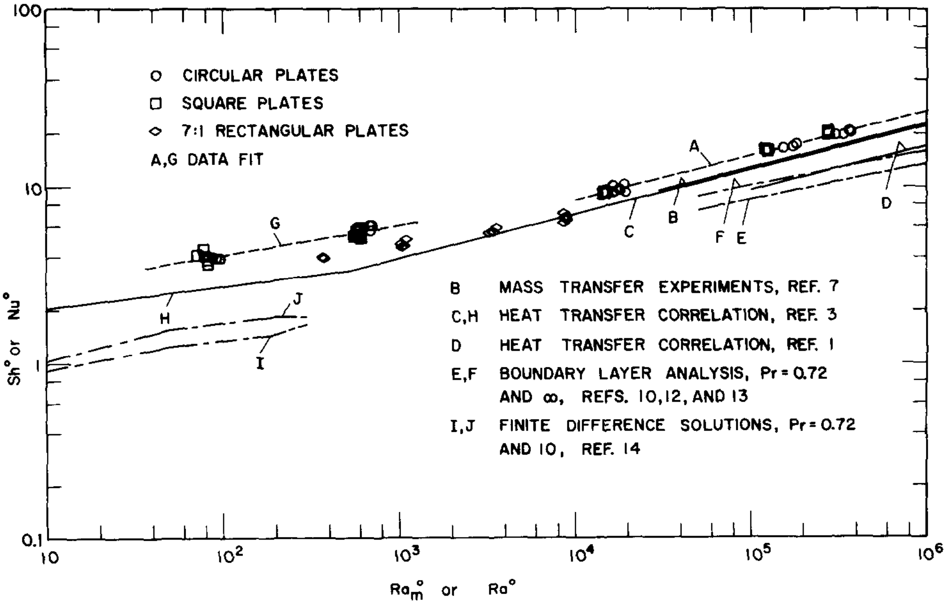

FIG. 2. Presentation of mass transfer and heat transfer results. Conventional characteristic lengths L^0 are employed in all dimensionless groups.

fied at the upper left of the figure, as are the dashed lines A and G that have been fitted through the data points for the circular and square plates. In addition to the present experimental results, the figure contains a number of curves representing the results of prior investigations. These curves are identified at the lower right of the figure, both as to reference source and to the circumstances under which they are obtained. The prior work cited in the figure includes mass transfer experiments, heat transfer correlations, and boundary layer and finite-difference solutions.

These results and those of the present experiments have been brought together on a single figure by invoking the analogy between heat and mass transfer. In discussing the figure, Sherwood number will be used interchangeably with Nusselt number, Schmidt number with Prandtl number, and mass transfer Rayleigh number with heat transfer Rayleigh number. The superscript 0 has been employed to indicate that the convectional characteristic lengths L^0 are contained in Sh, Nu, Ra and Ra_m.

Aside from the finite-difference results of [14], all of the lines appearing in Fig. 2 are represented by the relation

$$Sh^0 = a(Ra_m^0)^n \quad \text{or} \quad Nu^0 = a(Ra^0)^n. \qquad (9)$$

The constants a and n corresponding to the straight lines of Fig. 2 are listed in Table 1 along with other relevant information.

If attention is first turned to the upper end of the Rayleigh number range, it is seen from the figure and the table that all of the experimental results (curves A, B, C and D) are correlated by a $\frac{1}{4}$-power Sherwood–Rayleigh relation. The present data for circular and square plates (A) lie about 20 per cent above Mikheyev's correlation (C), the basis of which is, unfortunately, not described in [3]. The oft-quoted Fishenden and Saunders correlation (D) (e.g. see McAdams [18]) falls substantially lower. The present experimental results, taken together with those of Wragg and Loomba (B) support Mikheyev's correlation over that of Fishenden and Saunders. It may also be pointed out that the correlation equation of Bosworth [2] is numerically very close to Mikheyev's (C), but the characteristic dimensions in Nu and Ra are not clearly specified.

Table 1. Supplemental information for Fig. 2 and equation (9)

Line	a	n	Remarks
A	0·84	$\frac{1}{4}$	Expt., $Sc = 2·5$
B	0·72	$\frac{1}{4}$	Expt., $Sc = 2300$, circular plates
C	0·70	$\frac{1}{4}$	Expt., (?)
D	0·54	$\frac{1}{4}$	Expt., $Pr \sim 0·7$, square plates
E	0·841	$\frac{1}{5}$	Anal., $Pr = 0·72$
F	1·01	$\frac{1}{5}$	Anal., $Pr = \infty$
G	1·90	$\frac{1}{6}$	Expt., $Sc = 2·5$
H	1·53	$\frac{1}{8}$	Expt., (?)

Also shown in Fig. 2, in the upper end of the Rayleigh number range, are lines E and F which represent the predictions of laminar boundary layer theory for a highly elongated rectangular plate, that is, a plate of infinite length and finite width. These lines are based on numerical solutions of the ordinary differential equations which are derived from the governing conservation equations by means of a similarity transformation [10, 12, 13]. The lines E and F, which correspond to $Pr = 0·72$ and ∞ respectively, indicate a separate dependence on the Prandtl number of about 20 per cent over the range considered.

The analytical predictions (E, F) fall well below the present experimental results (A), as well as those of Wragg and Loomba (B), and Mikheyev (C). Furthermore, analysis gives a $\frac{1}{5}$-power dependence on the Rayleigh number, whereas the experimental results are all well correlated by a $\frac{1}{4}$-power dependence. As a consequence, at still higher Rayleigh numbers, the disparity between analysis and experiment would even be larger than that shown in Fig. 2. In addition to the just-mentioned analytical results, approximate results based on momentum and energy integrals are available [8, 11]. These integral solutions also lead to a $\frac{1}{5}$-power dependence on the Rayleigh number. The different slopes of the analytical and experimental results is an interesting finding that merits further exploration.

In the lower end of the Rayleigh number range of Fig. 2, Sh varies more slowly with Ra_m than in the already discussed upper end of the range. This behavior, which has been observed in other natural convection problems, is usually attributed to the disappearance of the boundary layer regime. The present data for circular and square plates fall together and are well correlated by a $\frac{1}{6}$-power dependence on the Rayleigh number (G). Mikheyev's correlation line H lies below the present data and has a different slope from line G. No assessment of the reasons for these differences can be made since the basis of Mikheyev's expression is not known. The finite difference results of Suriano and Yang [14], lines I and J, fall well below the experimental results.

Up to this point, no attention has been given to the data for the 7:1 aspect ratio rectangular plates. Whereas the circular and square plate Sherwood numbers were represented by a common correlation, those for the rectangular plates are quite separate. It is not at all surprising that a universal correlation is not achieved in Fig. 2, since the characteristic lengths appearing in Sh and Ra_m were not selected in a systematic manner. It will now be demonstrated that a more careful selection of the characteristic lengths will lead to a common correlation for all three of the plate geometries investigated here.

An alternative set of characteristic lengths, designated as L^*, was evaluated from

$$L^* = A/P \qquad (10)$$

where A is the plate surface area and P is the perimeter which encompasses the area. As will be discussed shortly, the lengths L^* appear to be related to the average horizontal distance traveled by fluid particles moving from the edges of the plate into the interior. Aside from a factor of four, equation (10) is reminiscent of the definition of the equivalent diameter.

From equation (10), the characteristic lengths L^* were evaluated as: (a) $\frac{1}{4}$ of the side of a square plate, (b) $\frac{1}{4}$ of the diameter of a circular

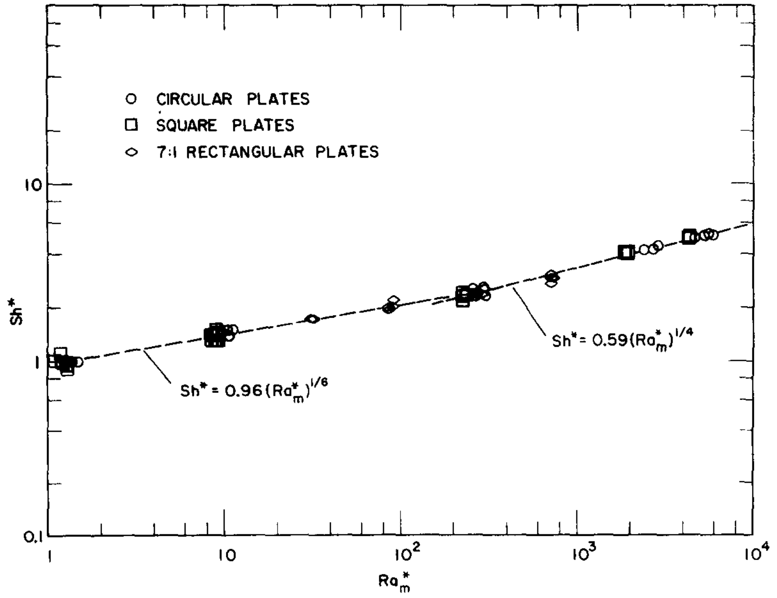

FIG. 3. Correlation of present mass transfer results in terms of Sherwood and Rayleigh numbers based on the characteristic lengths L^*.

plate, and (c) $(\frac{7}{8}) \times (\frac{1}{2})$ of the short side of a 7:1 rectangular plate. With these, the experimental data, expressed in terms of Sh^* and Ra_m^*, are plotted in Fig. 3. Inspection of the figure reveals that the experimental results for the three planforms now have a common correlation. Least-squares fits of the data yielded separate correlation equations for the ranges $Ra_m^* > 200$ and $Ra_m^* < 200$. The computer output from the least-squares routine gave values of 0·256 and 0·169 for the exponents of the Rayleigh number in the correlating equations for the two regimes. With no significant loss of accuracy, these exponents were taken as $\frac{1}{4}$ and $\frac{1}{6}$ and the least-squares routine reapplied to find the multiplicative constants. The corresponding correlation equations are

$$Sh^* = 0{\cdot}59(Ra_m^*)^{\frac{1}{4}}, \quad Ra_m^* > 200 \qquad (11)$$

$$Sh^* = 0{\cdot}96(Ra_m^*)^{\frac{1}{6}}, \quad Ra_m^* < 200. \qquad (12)$$

From an examination of the flow visualization photographs of [5], it appears that the lengths L^* of equation (10) closely approximate the average path lengths of particles moving from the edges of the plate into the interior. In fact, the authors' first correlations were in terms of these path lengths. The reason for favoring equation (10) over the path length concept is that it gives a direct, systematic way of evaluating characteristic lengths for geometries for which flow visualization information is not available. It is the expectation of the authors that the correlating equations (11) and (12) should be applicable to other planforms, with L^* evaluated in accordance with equation (10).

As a final matter, it remains to comment on the analogy between heat and mass transfer. In most mass transfer experiments, such as those performed here, there is a non-zero normal velocity at the plate surface. On the other hand, such a surface velocity is generally not present in corresponding heat transfer problems. To confirm the validity of the heat and mass transfer analogy, it is necessary to demonstrate that the influence of the surface mass transfer velocity is negligible.

On the basis of the measured sublimation rates, the surface mass transfer velocities were calculated to be on the order of 10^{-5}–10^{-6} m/s, which is surely very small. To estimate the possible influence of these small velocities on the Sherwood number results, use was made of the horizontal plate analysis of Kadambi [13]. The dimensionless surface velocity of Kadambi's analysis, f_w, was evaluated from the experimental measurements to be on the order of 10^{-4}. The smallest non-zero value of f_w for which Kadambi gives results is 10^{-3}. Interpolation in his Table 1 suggests that the surface velocities of the present experiments should have an altogether negligible influence on the results. Estimates were also made using the vertical plate solutions of Sparrow and Cess [19], and the same conclusion about the effect of the surface velocity was reached.

CONCLUDING REMARKS

The present experiments appear to be the first on natural convection mass transfer involving surface sublimation. On the basis of the experience gained herein, it may be concluded that this experimental technique can be used to advantage in other natural convection problems.

The experimental results suggest that a common correlation for horizontal plates of different planforms can be attained if appropriate characteristic lengths are used in the Sherwood and Rayleigh numbers. For the circular, square, and 7:1 rectangular plates of this investigation, a common correlation was attained with characteristic lengths evaluated as the ratio of the surface area to the encompassing perimeter. It is the expectation of the authors that the correlation will be applicable to other planforms, provided that the characteristic lengths are chosen in the manner just discussed.

The present results, together with already published electro-chemical mass transfer results, tend to support the correlation of Mikheyev over the oft-quoted Fishenden and Saunders correlation. For the most part, the presently available experimental data lie well above the predictions of boundary layer theory. Furthermore, the experimentally determined transfer coefficients vary with the $\frac{1}{4}$-power of the Rayleigh number, whereas boundary layer theory gives a $\frac{1}{5}$-power dependence. In the range of lower Rayleigh numbers, the transfer coefficients vary more slowly with the Rayleigh number. In this range, the transfer coefficients from the experiments are very much higher than those given by a numerical finite-difference solution.

ACKNOWLEDGEMENT

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TRANSFERT MASSIQUE PAR CONVECTION NATURELLE ADJACENTE A DES PLAQUES HORIZONTALES

Résumé—Des expériences sur la convection naturelle adjacente à des surfaces planes horizontales ont été conduites par utilisation de la technique de sublimation de naphtalène. Par analogie, les présentes expériences de transfert massique correspondent au transfert thermique à une plaque à surface orientée vers le haut et chauffée isothermiquement ou à une plaque refroidie à surface orientée vers le bas. On a employé dans les essais différentes formes planes, circulaires, carrées et rectangulaires 7 : 1. Des coefficients globaux de transfert massique et des nombres de Sherwood ont été déterminés et reliés en fonction du nombre de Rayleigh. On a abouti à une relation commune pour les trois formes planes en utilisant les longueurs caractéristiques égales au rapport de l'aire au périmètre. Les coefficients de transfert estimés par la théorie de la couche limite laminaire sont au-dessous des résultats expérimentaux et montrent une dépendance différente du nombre de Rayleigh. On a trouvé que la formule empirique souvent mentionnée de Fishenden et Saunders est une représentation moins satisfaisante des résultats présents que celle de Mikheyev. Dans le domaine des faibles nombres de Rayleigh les résultats donnés par une solution numérique aux différences finies sont beaucoup plus faibles que les résultats expérimentaux.

STOFFÜBERGANG DURCH FREIE KONVEKTION AN HORIZONTALEN PLATTEN

Zusammenfassung—Es wurden Versuche über die freie Konvektion an horizontalen, ebenen Oberflächen mit Hilfe einer Naphthalinsublimationstechnik durchgeführt. Die vorliegenden Stoffübergangsversuche stehen in Analogie zum Wärmeübergang an isothermen, nach oben zeigenden beheizten oder nach unten weisenden gekühlten Flächen. Die Versuche wurden an kreisförmigen, quadratischen und rechteckigen (Seitenverhältnis 7:1) Ebenen durchgeführt. Mittlere Stoffübergangskoeffizienten und Sherwood-Zahlen wurden bestimmt und als Funktion der Rayleigh-Zahl korreliert. Eine für alle drei Ebenen gültige Korrelation wurde durch Verwendung charakteristischer Längen gefunden, die gleich dem Verhältnis Oberfläche zum Umfang sind. Die Übergangskoeffizienten, die sich aus der laminaren Grenzschichttheorie ergeben, fallen gut unter die experimentellen Daten, sie weisen eine anders geartete Abhängigkeit von der Rayleigh-Zahl auf. Die oft zitierte Korrelation von Fishenden und Saunders gibt, wie sich zeigt, die vorliegenden Daten weniger befriedigend wieder als die von Mikheyev. Im Bereich kleiner Rayleigh-Zahlen liegen die Ergebnisse eines numerischen Differenzenverfahrens sehr viel tiefer als die experimentellen Daten.

ПЕРЕНОС МАССЫ НА ГОРИЗОНТАЛЬНЫХ ПЛАСТИНАХ ПРИ НАЛИЧИИ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ

Аннотация—С помощью методики, основанной на сублимации нафталина, проводились экспериментальные исследования по естественной конвекции на плоских горизонтальных поверхностях. Опыты, проводимые по переносу массы, аналогичны опытам по переносу тепла на изотермической нагреваемой пластине, обращенной поверхностью вверх, либо на охлаждаемой пластине, обращенной поверхностью вниз. В опытах использовались круглая и квадратная пластины, а также прямоугольная пластина с соотношением сторон 7:1. Суммарные коэффициенты переноса массы и значения числа Шервуда были определены и обобщены как функция числа Релея. Общая зависимость для всех трех пластин получена с помощью характерных длин, равных отношению площади поверхности к её периметру. Коэффициенты переноса, рассчитанные по теории ламинарного пограничного слоя, ложатся значительно ниже экспериментальных данных и обнаруживают иную зависимость от числа Релея. Найдено, что часто цитируемое эмпирическое соотношение Фишендена и Саундерса хуже удовлетворяет имеющимся данным, чем аналогичное соотношение Михеева. В диапазоне более низких значений числа Релея результаты численных конечно-разностных решений намного ниже экспериментальных данных.